Randomized Benchmarking with Confidence

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QIP 2015
13 January 2015
Sydney

Joint work with Joel Wallman

arXiv:1404.6025
New J. Phys. **16** 103032 (2014)

Fault-tolerance threshold theorem

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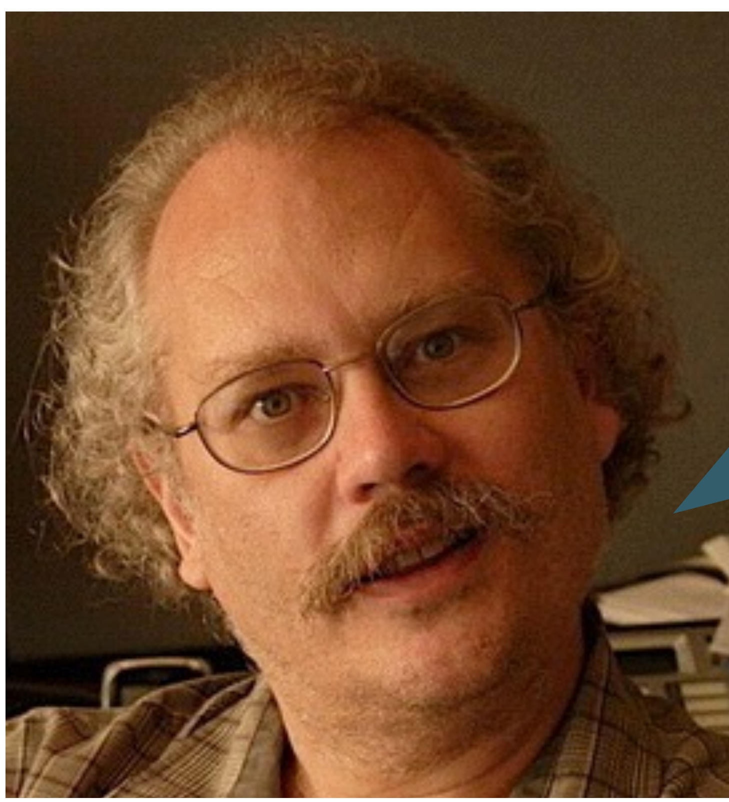
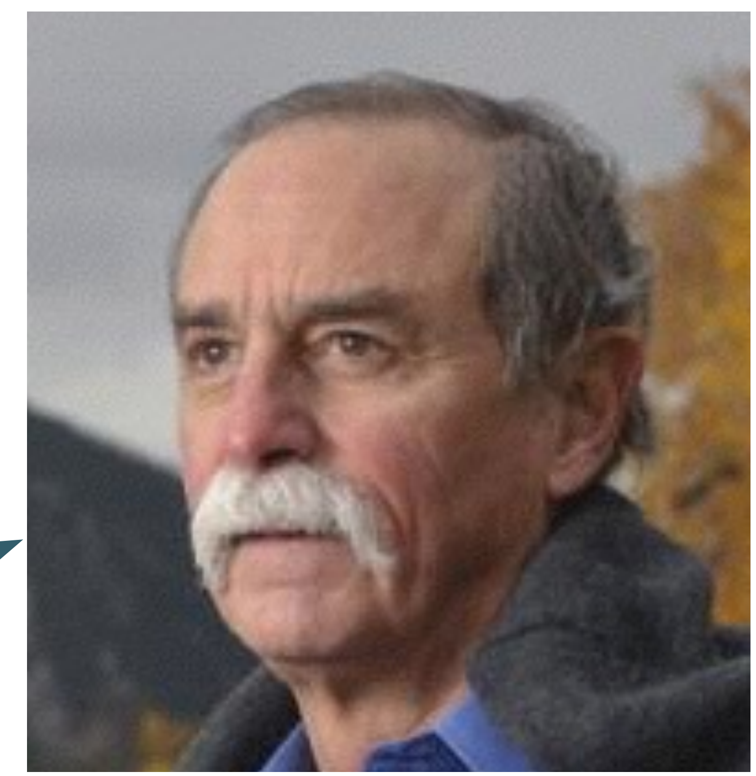


Where's my quantum computer?

I'm workin' on it!



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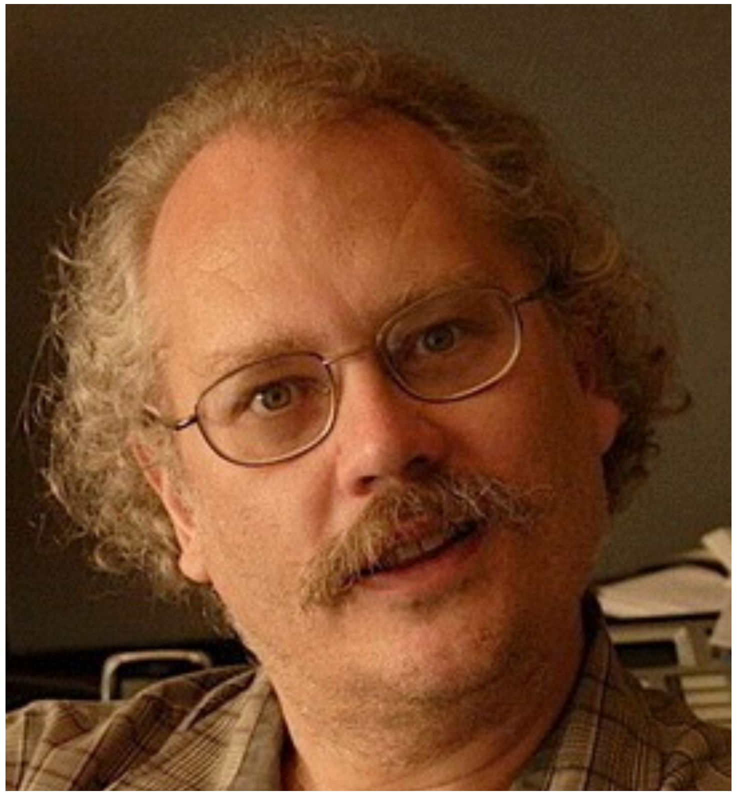
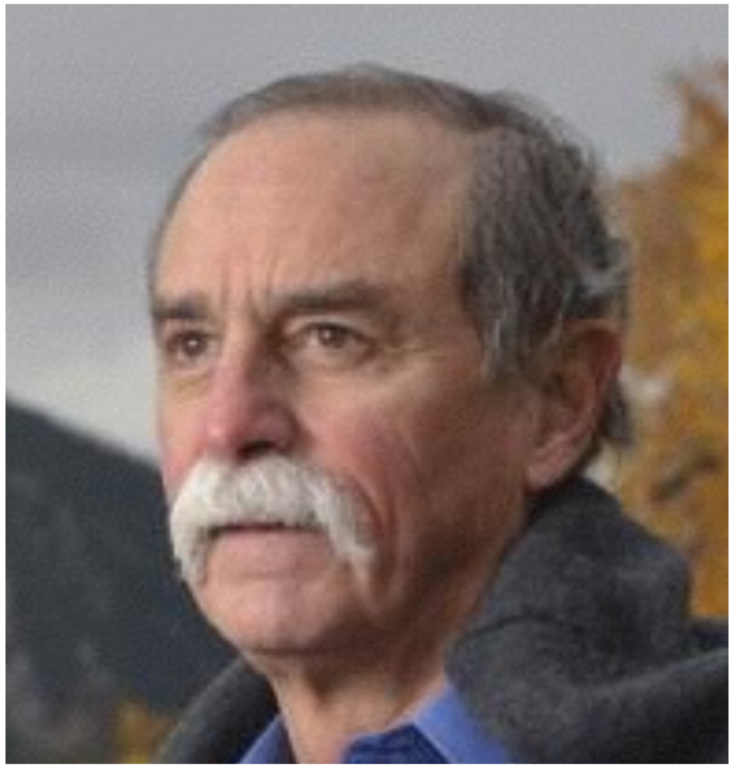


- Even with really good **mustaches**, it's tough!
- ε is pretty small, ~0.1% or less.
- the overhead is still quite demanding



Is our computers working?

How can we check if our system is FTTT compliant when we are near the threshold?

- Overhead prohibits a direct demonstration even if you are just under the threshold
- High-precision demands and $O(1/\varepsilon^2)$ scaling of sampling methods introduce challenges

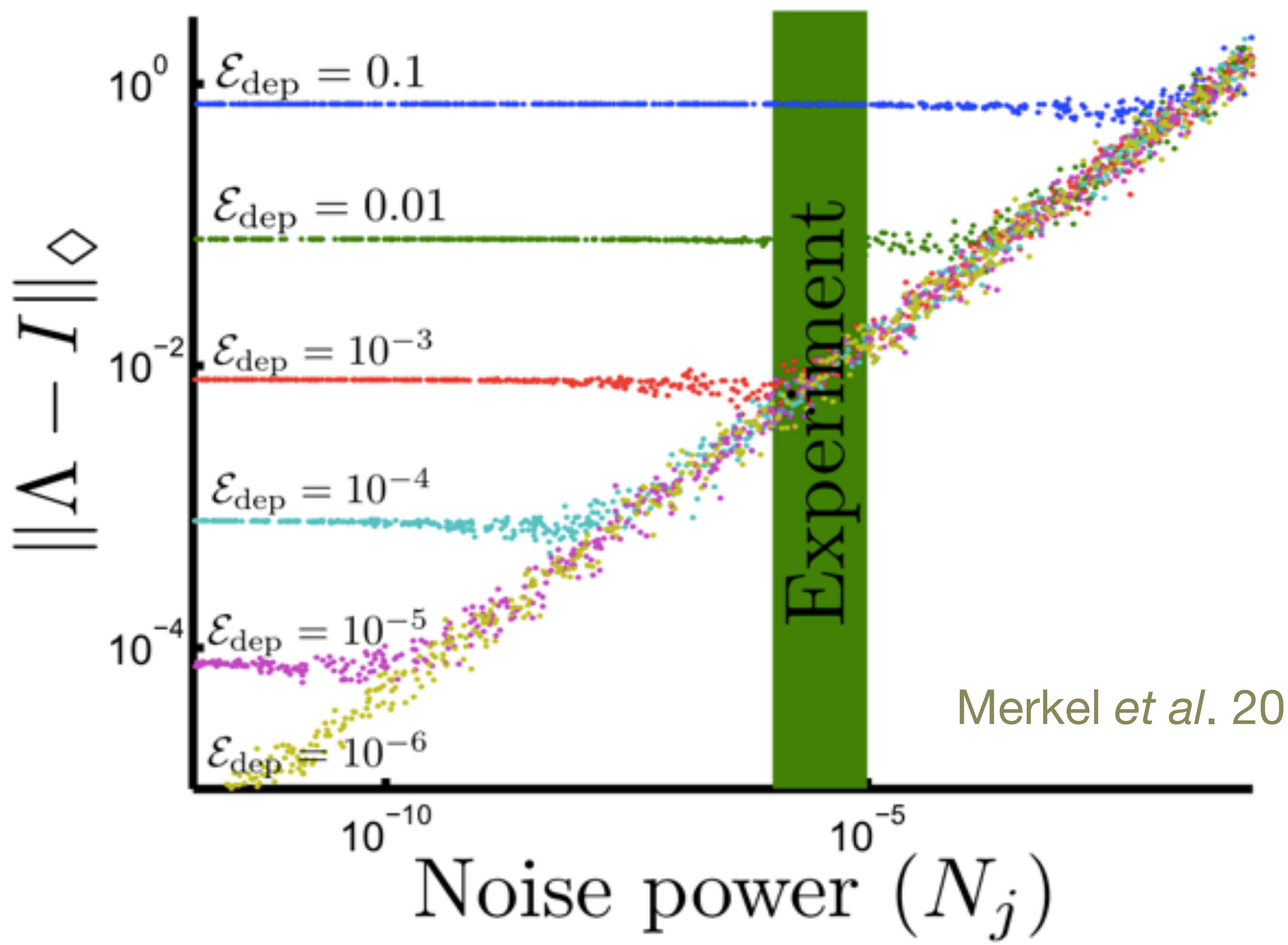
Complexity is a **practical** bottleneck for assessing quantum devices

What do we really want?

- We want to evaluate progress toward the goal of FTQC
- Experimentalists (and funding agents!) want **standardized numbers** that are comparable across radically different platforms
- The numbers should be related to something **operational**, e.g.
  - **trace distance** for states (worst case)
  - **diamond distance** for channels (worst case)
  - **gate fidelity** (average case)
- **Complexity** of obtaining a useful estimate should not be prohibitive on multi-qubit systems (the more the better)

State and Process Tomography

Fine in principle, but fails in practice due to the inevitable presence of a noise floor



Merkel *et al*. 2013.

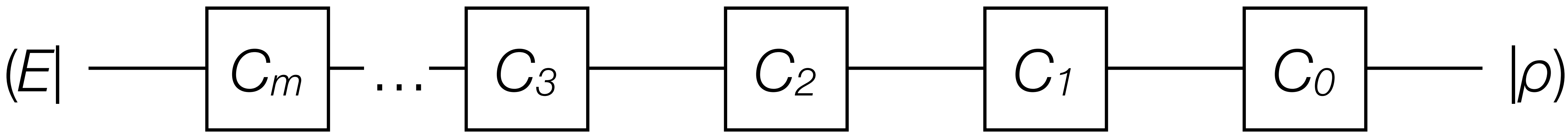
SPAM Errors

State Preparation And Measurement Errors

Some SPAM-resistant tomographic methods are being developed: Blume-Kohout *et al*. 2013; Kimmel *et al*. 2014.

Randomized Benchmarking

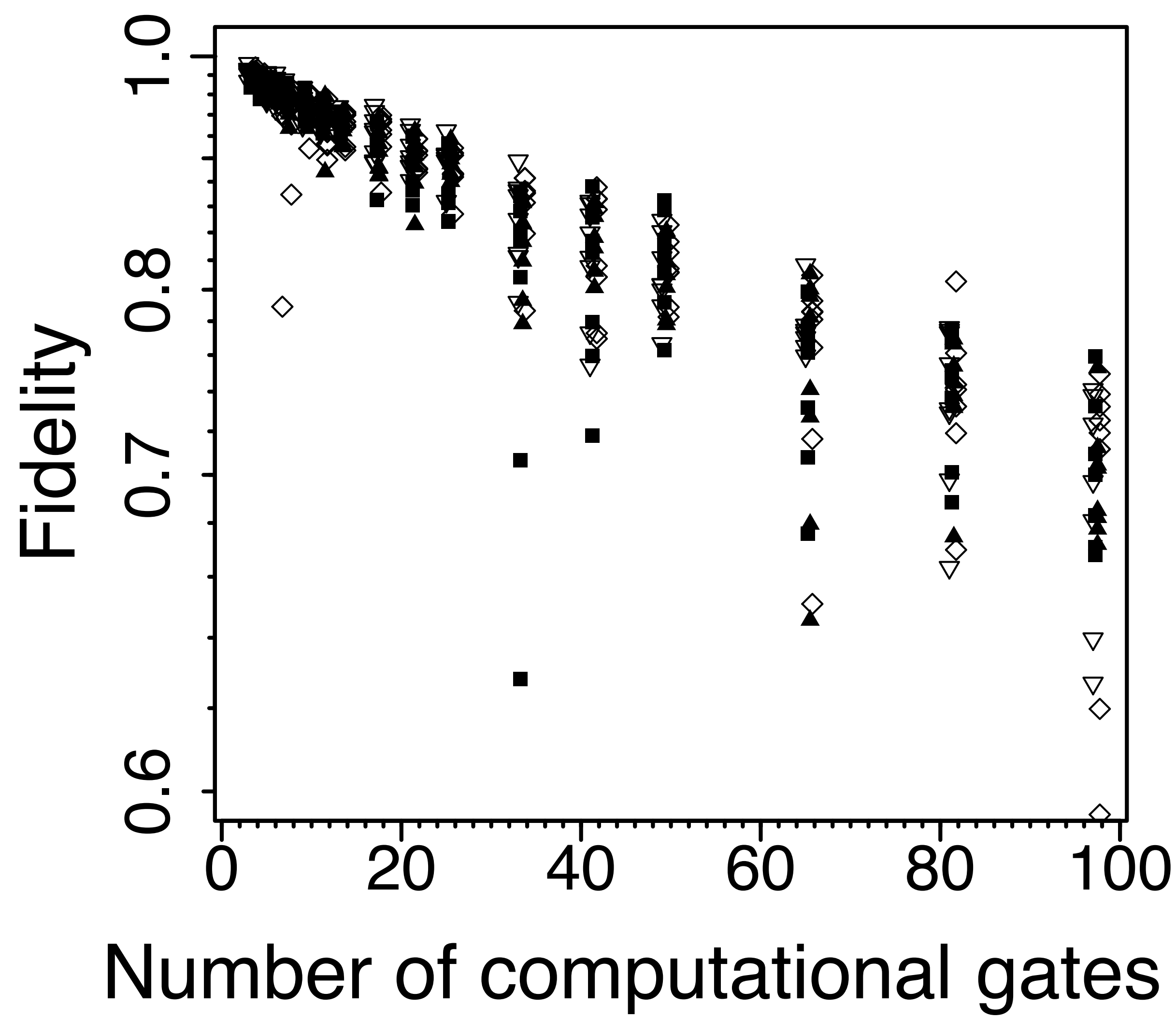
Emerson, Alicki, Zyczkowski 2005; Knill *et al*. 2008.



- Choose a random set *s* of *m* Clifford gates
- Prepare the initial state in the computational basis
- Apply the Clifford sequence, and add the inverse gate at the end of the sequence
- Measure in the computational basis

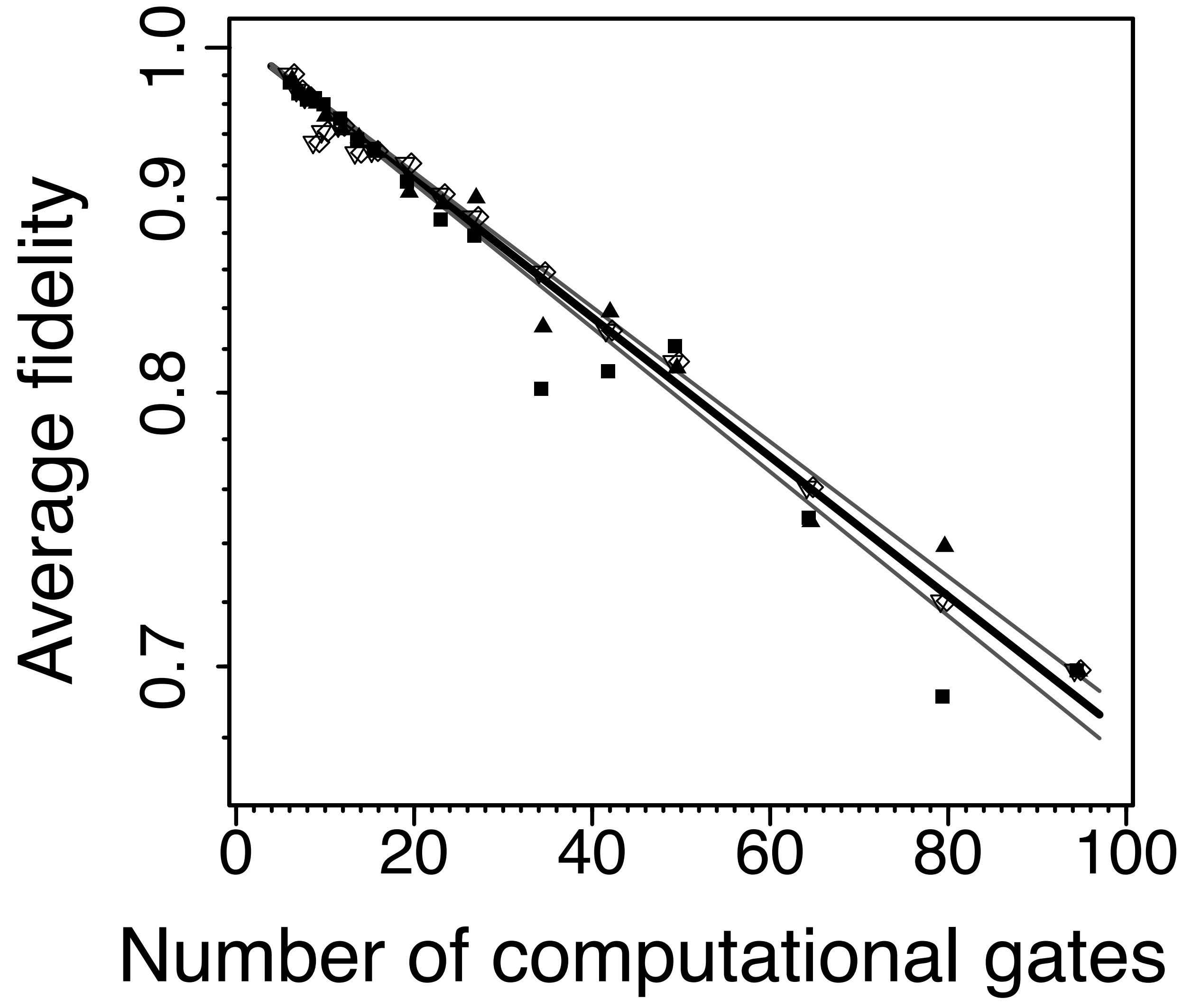
Repeat to estimate $F_{m,s} = \Pr(E|s,\rho)$

Randomized Benchmarking



Knill *et al*. 2008.

Randomized Benchmarking



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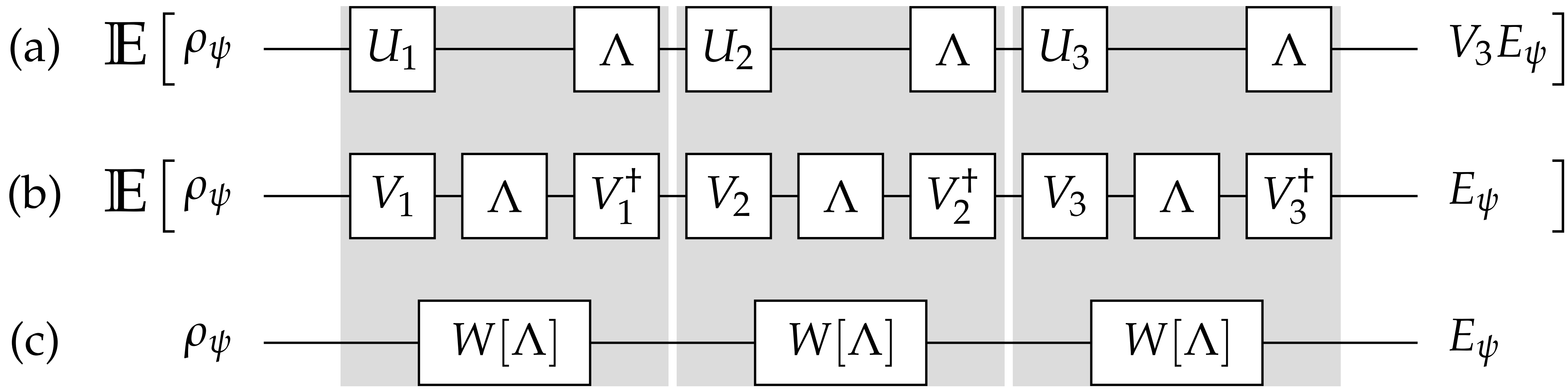


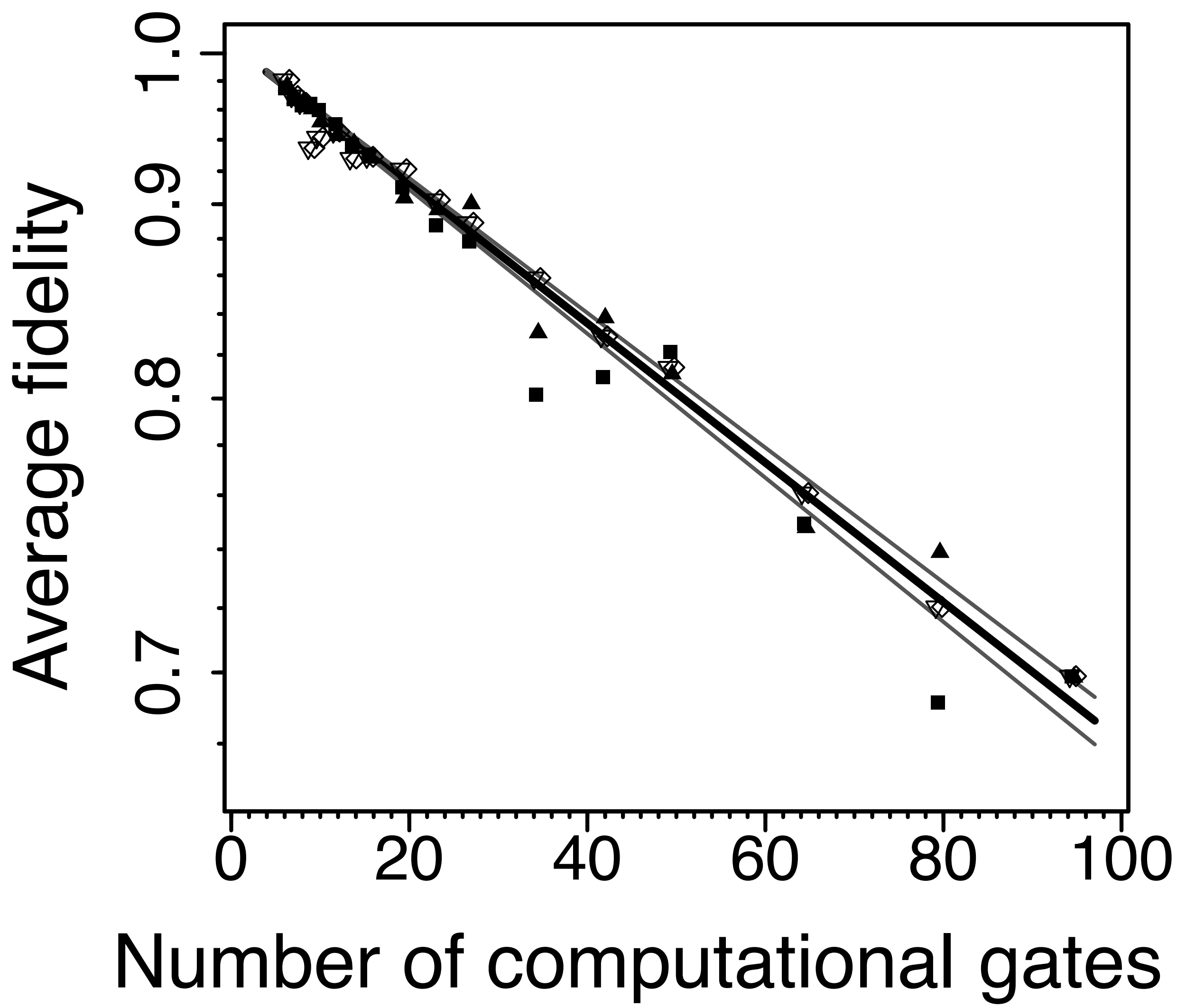
Magesan, Gambetta, & Emerson 2012; Granade, Ferrie, & Cory 2014.

$$
\begin{aligned}
W[\Lambda](\rho) &= \int \mathrm{d}U\, U^\dagger \Lambda[U\rho U^\dagger] U \\
&= \frac{1}{|\mathcal{C}|} \sum_{U\in\mathcal{C}} U^\dagger \Lambda[U\rho U^\dagger] U \\
&= f\rho + (1-f)\frac{\mathbb{1}}{d}
\end{aligned}
$$

$$
f = \frac{d\mathcal{F}_{\text{avg}}(\Lambda) - 1}{d-1}
$$

$$
\mathcal{F}_{\text{avg}}(\Lambda) = \int \mathrm{d}\psi \operatorname{Tr}[\psi\Lambda(\psi)]
$$

Randomized Benchmarking



Knill *et al*. 2008.

"0th order model":
Fit to the model

$$\bar{F}_m = A + B f^m$$

Note this is **not** a linear model!

Optimal Experiment Design

Given some prior knowledge about our average error rate (e.g. from Rabi oscillations), how can we design an optimal experiment?

We need to know how the variance changes with m and $r = 1\text{-}F_{\text{avg}}$.

Unfortunately, naive bounds depend on the SPAM:

$$\sigma_m^2 \leq (A+B)(1-A-B) + \frac{mdBr}{d-1} + O(m^2 r^2)$$

This leads to estimates of sampling ~10^5 sequences!

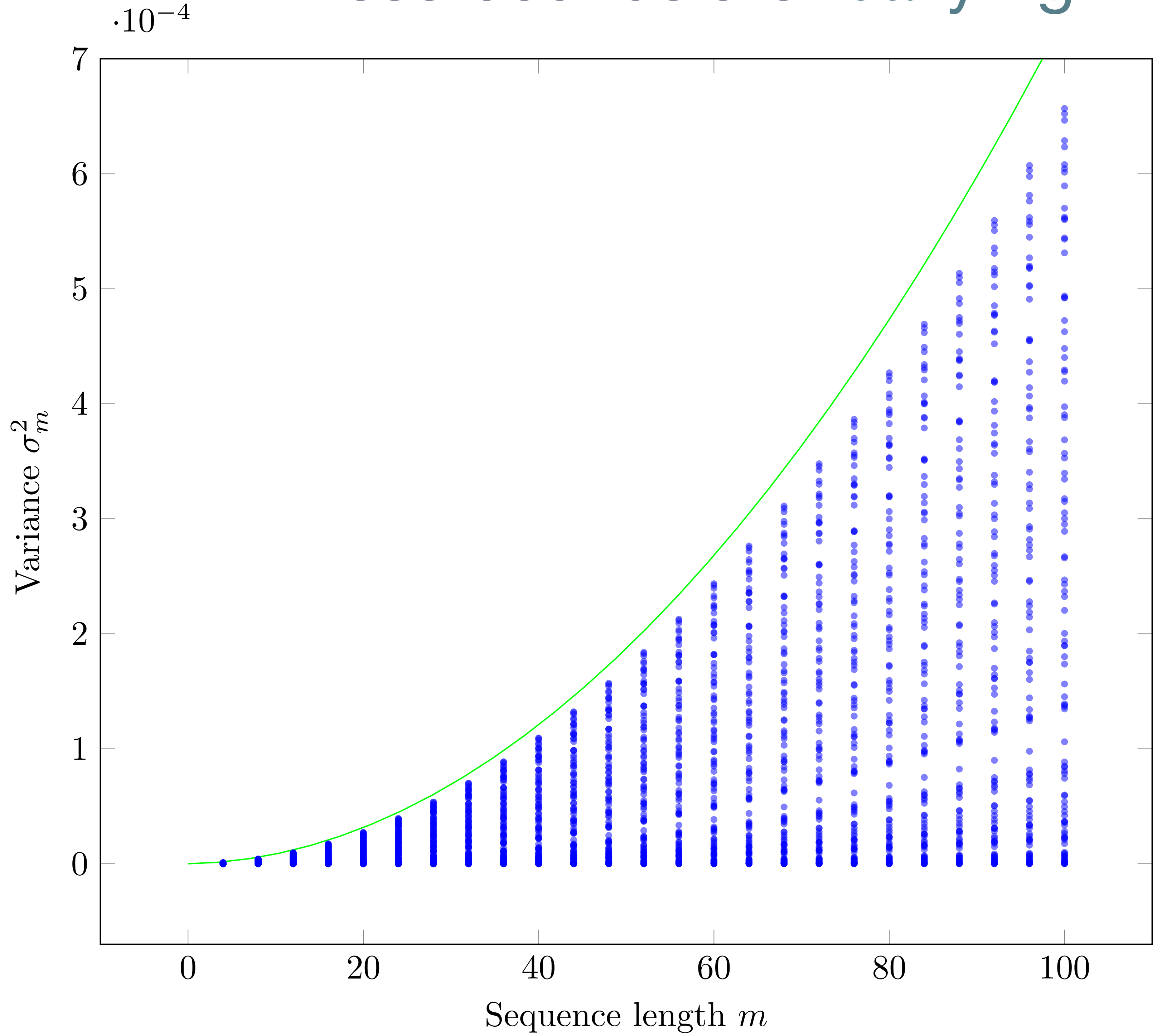
Epstein *et al*. 2014

Our Contribution

Wallman & STF 2014

- Reduce the variance bound from O(1) to O(*mr*)
- For general *d*-level systems, the bound is
$$\sigma_m^2 \leq 4d(d+1)mr + O(m^2r^2d^4)$$
- For the special case of qubits, we obtain
$$\sigma_m^2 \leq m^2r^2 + \frac{7mr^2}{4} + 6\delta mr + O(m^2r^3) + O(\delta m^2r^2)$$
- If the noise is diagonal in the Pauli basis (e.g. depolarizing or dephasing noise), we obtain
$$\sigma_m^2 \leq \frac{11mr^2}{4} + O(m^2r^3)$$
- Plus some robustness guarantees against weak time-dependent and nonmarkovian noise…

These bounds are really tight!



This result leads to estimates on the order of 100 sequences compared to previous estimates of ~10^5 sequences

Random channels sampled using Ruskai, Szarek, & Werner 2002

Our Methods

- Variance depends on the average of the tensor power

$$\sigma_m^2 = (E^{\otimes 2}|\Big(\big[(\Lambda^{\otimes 2})^{\mathcal{G}}\big]^m - \Big[\big(\Lambda^{\mathcal{G}}\big)^{\otimes 2}\Big]^m\Big)|\rho^{\otimes 2})$$

- Use plethysm of the Clifford group; Schur's lemma is not enough!

$$(\Lambda^{\otimes 2})^{\mathcal{C}_2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \varphi^{\mathcal{C}_2} & 0 & 0 \\ 0 & 0 & \varphi^{\mathcal{C}_2} & 0 \\ P_1\alpha^{\otimes 2} & |\mathcal{C}_2|^{-1}\sum_{g\in\mathcal{C}_2} g\alpha\otimes\varphi^{(g)} & |\mathcal{C}_2|^{-1}\sum_{g\in\mathcal{C}_2}\varphi^{(g)}\otimes g\alpha & (\varphi^{\otimes 2})^{\mathcal{C}_2} \end{pmatrix}$$

Our Methods

- A good bound requires a very delicate cancellation

$$(\Lambda^{\otimes 2})^{\mathcal{G}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & f\mathbb{1} & 0 & 0 \\ 0 & 0 & f\mathbb{1} & 0 \\ P_1\alpha^{\otimes 2} & b & c & \left(\varphi^{\otimes 2}\right)^{\mathcal{G}} \end{pmatrix} \qquad (\Lambda^{\mathcal{G}})^{\otimes 2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & f\mathbb{1} & 0 & 0 \\ 0 & 0 & f\mathbb{1} & 0 \\ 0 & 0 & 0 & f^2\mathbb{1} \end{pmatrix}$$

- Need to bound the spectral gap of the averaged tensor power of the transfer matrix
- Analysis proceeds by bounding the nonunital contribution, von Neumann's trace inequality, and a lot of sweat.

A Conjecture

Definition 15. A channel $\Lambda : \mathcal{D}_d \to \mathcal{D}_d$ is *n-contractive* with respect to a group $\mathcal{G} \subseteq \mathsf{U}(d)$ if $(\Lambda^{\otimes n})^{\mathcal{G}}$ has at most one eigenvalue of modulus 1.

We conjecture that **all** nonunitary channels are 2-contractive with respect to any unitary 2-designs

Proposition 16. *Let Λ be a completely positive, trace-preserving and unital channel and $\mathcal{G}$ a unitary 2-design. Then Λ is 2-contractive with respect to $\mathcal{G}$ if and only if it is nonunitary.*

An equivalent statement: the averaged tensor power channel is "**strongly irreducible**" whenever the channel is nonunitary

This result guarantees that the asymptotic variance decays exponentially to a fixed constant that depends only on the magnitude of the nonunital part of the channel.

Sanz, Perez-Garcia, Wolf, & Cirac 2010

Conclusions & Open Questions

- Showed a rigorous bound on the RB variance at $O(mr)$
- Go beyond 0th order approximation
- Remove the dimensional factor for general *d*
- Plethysm of the Clifford irrep for higher *d*
- Is a closed-form solution possible for qubits?
- Improve our mustaches to do better science?
- See *arxiv:1404.6025* (NJP 2014) for more details!